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Casimir-like effect on a granular pile

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We investigate experimentally a Casimir-like effect in a three-dimensional pile of rice, which has a power-law avalanche size distribution. We observe the change in distance between two Plexiglas sheets placed on the pile parallel to each other and parallel to the mean avalanche flow direction, while rice grains are continuously and uniformly falling on top of the pile. The resulting avalanches are fluctuations, confinement of which is found to drive the two plates together. During 25-h experimental runs, for initial intersheet distances ranging from 20.0 to 90.0 mm we observe changes in the range from 6.0 mm to less than 1.0 mm. A similar distance dependence is obtained from a simple analytical model.

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I. INTRODUCTION

Casimir derived [1] in 1948 that an attractive force between two metal plates in vacuum should exist due to vacuum fluctuations of electromagnetic waves. The magnitude of the Casimir force is directly related to the cross-sectional area and is inversely proportional to the fourth power of the separation distance between the plates. The Casimir force has also been investigated for other materials and geometries [2]. Even repulsive interactions have been predicted [3]. With the recent developments in microtechnology and nanotechnology, several experimental techniques have been realized to investigate the Casimir effect. One of those experiments has measured Casimir force values in the order of micronewtons with the interaction distance in the micrometer range and in vacuum [4]. Recent research indicates that the Casimir effect can still be the dominant interaction between parallel surfaces even at atmospheric pressure and room temperature. In such experiments, the Casimir force can be changed by varying the dielectric properties of the interacting materials [5]. A measured Casimir force between two gold spheres immersed in ethanol is smaller than the value predicted in vacuum [6] and can be repulsive with certain optical properties of the material, as first measured in [7] and followed up in [8]. The understanding of such effects is important for the behavior of nanosystems and microsystems (e.g., MEMS) immersed in air or fluids.

Since the Casimir effect is due to electromagnetic fluctuations, it seems plausible that confinement of other fluctuating fields could give rise to a similar effect. Indeed in colloids immersed in a binary liquid mixture [9], the classical thermal fluctuations in the surrounding medium give rise to a so-called critical Casimir effect. The temperature dependence of this effect has been investigated in [10,11].

A Casimir-like effect between spheres and flat plates in granular fluids has also been proposed [12,13], where the fluctuations are in the form of a Gaussian white noise introduced into the granular system [14]. Numerical simulations reveal that there can be a long-range repulsive interaction between two large spheres immersed in a sea of small particles [12,13].

In this paper, we show that another fluctuating system where Casimir-like effect can be observed is a granular pile, which is a self-organized critical (SOC) system [15] with spontaneous fluctuations in the form of avalanches. Such nonequilibrium systems are characterized by fluctuations on all scales and stay critical without external tuning. Although the sand pile as paradigm for such behavior has been criticized, it has been demonstrated that an avalanching rice pile [16,17] shows many SOC properties, including a power-law distribution of avalanche sizes. Here, we investigate a Casimir-like behavior due to the avalanches on two parallel Plexiglas sheets inserted in a rice pile.

In Sec. II details of the experimental procedure are given. In Sec. III we present results of approximately 40 experiments with total duration of more than 1000 h and investigate the dependence of the magnitude of the Casimir-like force on the initial distance between the Plexiglas sheets. In addition, we correlate positive and negative changes in the sheet distance to avalanche events and the shape of the avalanches. In Sec. IV we present a simple model for this distance dependence, and in Sec. V we discuss the similarities and discrepancies between our experiment and other Casimir-like systems.

II. EXPERIMENTAL DETAILS

A. Initialization of the experiment

Figure 1 shows a schematic of the rice pile in a half-open box (~ 1 m^3) with the two transparent Plexiglas sheets inserted near the center of the pile surface. The sheets, each with dimensions of $294\times 210\times 1$ mm^3, are placed parallel to each other at an initial separation distance D_i. A quarter of the height of the sheets (~ 50 mm) is initially buried, with the sheets placed parallel to the mean avalanche flow and perpendicular to the pile surface. The sheets are not fixed but are held in place by rice grains only. The pile is initially prepared close to the critical angle of $\sim 30°$ relative to the base. During the experiment, rice falls continuously and uniformly at the top of the pile at a rate of ~ 38 grains per second which is within the slow-driving range [18]. During the entire experiment, the foot of the rice pile rests on the horizontal plane of the box and never comes close to its open edge. A detailed description of the three-dimensional rice pile-avalanche system was previously published [18,19].

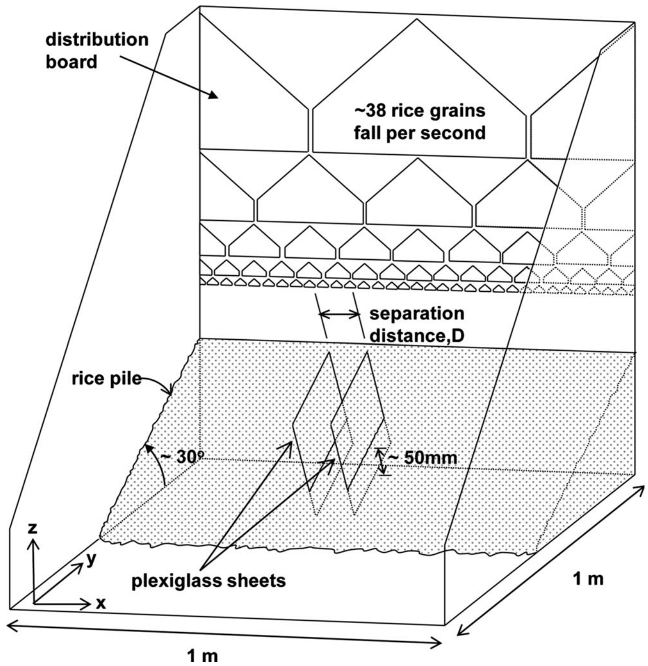

FIG. 1. Schematic of the experimental setup with the rice pile and the Plexiglas sheets. Rice enters the distribution board at the top, uniformly falls across the whole width of the top of the pile, and close to the back plane of the half-open box.

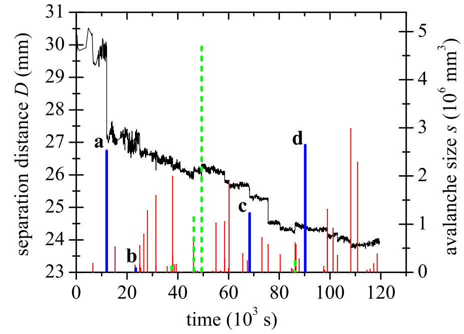

FIG. 2. (Color online) The average separation distance D between the two Plexiglas sheets (trace and left-hand axis) for a selected experiment shows a 20% decrease in D after 10^5 s. The vertical lines indicate occurrence and size of the avalanches (right-hand axis). Pictures of the avalanches (a)–(d) (blue solid lines) are shown in Fig. 3. A few avalanches (green dashed lines) result in a slight increase in D, while most (blue and red solid lines) result in a decrease in D.

B. Measurement procedure

During the experiment we determine (1) the three-dimensional surface topography of the rice pile and (2) the distance between the sheets. By subtracting the shape of the pile before and after an avalanche, the full three-dimensional shape of the avalanche is determined.

The three-dimensional shape of the rice pile is determined by monocular stereoscopy, using a pattern of red-green-blue lines projected onto the pile [18,19]. A charge-coupled device color camera with a resolution of 2560×1920 pixels captures images of the pile at an exposure time of 800 ms approximately every 15 s. Pictures of the rice pile are taken for approximately 10^5 s (over 25 h). The same pictures are analyzed by a computer program. One algorithm is used to deduce the three-dimensional rice pile surface topography and the avalanche size s (the volume of displaced grains) as described in [18,19]. The other algorithm finds the top of both sheets (in the z direction; see Fig. 1) and from that determines the distance between the sheets along x and averages over the sheets length in the y direction to obtain the separation distance D. To determine the dependence of the change in D on the initial separation, this procedure is repeated for different initial separation distances D_i ranging from 20.0 to 90.0 mm.

III. RESULTS AND DISCUSSIONS

A. Decrease in D with time and correlation between sheet motion and avalanches

The separation distance between the sheets during a single experiment is given in Fig. 2. A total decrease in D of approximately 20% is observed, from an initial value of 30.0 mm to a final one of around 24.0 mm. The distance decrease occurs mainly in abrupt changes (or jumps) due to the interaction of the sheets with avalanches. We note that the decrease in D is not monotonic: there are occasional increases.

To investigate how the fluctuations on the pile affect D, the avalanches are also indicated in Fig. 2 (the vertical lines mark their sizes). For about 93.5% of the avalanches (indicated by the red solid lines), a decrease in D is observed. However, a few avalanches lead to an increase in D (indicated by the green dashed lines), with a change of 0.5 mm or less. The largest size avalanche (the highest vertical green dashed line) increased D by 0.3 mm only. The blue lines [with labels (a)–(d)] indicate the size of the avalanches shown in Fig. 3.

Clearly avalanches of many magnitudes have occurred in this single 30-h experimental run. Small and large avalanches happen at both early and later time intervals. The effect on the relative motion of the sheets differs at either interval. At the beginning of the experiment, even small-sized avalanches result in a significant decrease in D. This is due to the sheets being less buried and more mobile with relatively less grains in the pile. On the other hand, at the later stage of the experiment, D does not decrease significantly even for large avalanches because the sheets are mostly buried (at least 75% of the sheet) in the rice grains and are less perturbed by motions in the pile. This is detrimental in determining the exact size of the Casimir-like force fluctuations in our experiment. However, it is still possible to establish the existence of such an effect and even to study its dependence on initial separation D_i, with similar experiments performed at different D_i's for the same duration and same amount of grains falling onto the pile.

Examples of some selected avalanche events (corresponding to the blue lines in Fig. 2) are illustrated in Fig. 3. Labels (a)–(d) in Fig. 2 correspond to the labels of the images in Fig. 3. In the latter figure, images represent the change in the pile due to an avalanche. Blue color (in the top of the image) indicates removal and red (lower in the image) indicates ad-

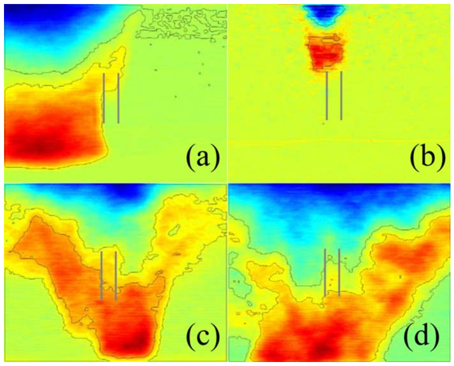

FIG. 3. (Color online) Pictures showing the displacement of grains during an avalanche. Blue (upper dark gray part of the image) indicates regions where grains are removed, and red (lower part of the image) indicates regions where grains are added. The saturation is a measure of the change in vertical height. Avalanches thus move rice grains from blue to red region. The positions of the sheets are indicated by vertical gray lines. (a) Avalanche reaching the sheets mainly on one side results in significant decrease in D. (b) Small avalanche reaching the top of the sheets causes sheets to move slightly apart. (c) Big avalanche that occurs slightly asymmetrically to the sheets results in a decrease in D. (d) Big avalanche reaching regions between and outside the sheets does not significantly change D (this may be enhanced because this avalanche occurred late in the experiment).

dition of grains due to the avalanche. Saturation is proportional to the local change in height of the pile. Hence, grains in an avalanche have moved from upper blue to lower red regions. The locations of the Plexiglas sheets are indicated by the two parallel gray lines. The big size avalanche [Fig. 3(a)] caused a big decrease in D while the small avalanche [Fig. 3(b)] reached the top of (and space between) the sheets and caused a slight increase in D by about 0.5 mm. For two big avalanches [Figs. 3(c) and 3(d)] (which both reached the space between and outside the sheets), the effects differ. Avalanche (c) moved the sheets closer together by 0.5 mm, while avalanche (d), although big in size, did not change the distance between the sheets. One possible reason is that this avalanche was already much broader than the sheet region before arriving there, so that it passed equally between and next to the sheets. Another reason is that it occurred rather late in the experiment, when the sensitivity is reduced due to burying of the sheets. Overall it is seen from Fig. 2 that changes in D happen due to the avalanches, but there is no direct correlation between the avalanche size s and the magnitude of change in distance ΔD.

In Fig. 4 more experiments similar to the one in Fig. 2 are shown for the same initial D_i of around 30.0 mm. Clearly all these experiments behave in a similar fashion, with (mainly downward) steps, and the final D's are roughly equal. Although the detailed dynamics is different, as expected for punctuated behavior, the overall behavior, i.e., a net motion bringing the sheets closer together, is the same. For all these experiments we observe a behavior consistent with a net attractive force originating from the fluctuating fields due to avalanches in the rice pile. In the next section we discuss the dependence on the initial separation distance D_i.

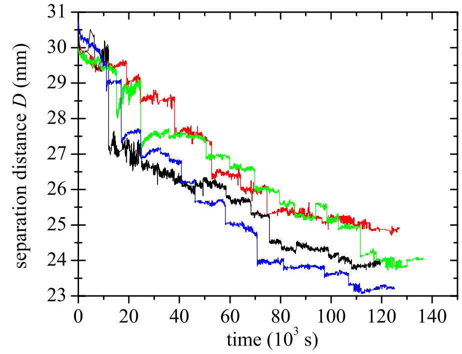

FIG. 4. (Color online) The temporal behavior of D for four different experiments. For all runs the initial separation is approximately 30 mm. Clearly, D is decreasing with time.

B. Dependence of the change in D on its initial value D_i

The conventional Casimir force between metallic plates separated by a distance D is proportional to D^{-4}, i.e., a very strong increase for decreasing distance. Qualitatively, one expects similar behavior for Casimir-like effects, because the smaller D is, the more fluctuating modes are suppressed, creating a larger imbalance in the forces from either side on a single sheet.

To investigate the distance dependence of the Casimir-like effect in the rice pile, experiments are performed for different initial separation distances D_i ranging from 20.0 to 90.0 mm. A plot of the change in the separation D averaged over typically five experiments versus the initial value D_i is shown by the curve in Fig. 5. A maximum change of 6.0 mm in the mean over five experiments is observed for sheets that are initially 30.0 mm apart. This change decreases to a minimum value of only 1.0 mm as the initial D increases to 90.0 mm. This behavior implies that the attractive interaction between the sheets weakens for larger separations. Since some of the experiments for $D_i=90.0$ mm resulted in a distance decrease close to zero or even a slight increase, we conclude that for distances larger than 90.0 mm, the net force acting on the sheets is fluctuating around zero.

Experiments with $D_i<20.0$ mm were not performed, because we needed to stay well above the length of individual

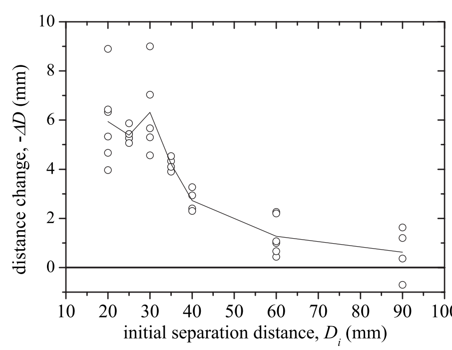

FIG. 5. Distance dependence of the Casimir-like force, shown as decrease in D versus initial separation distance D_i after 10^5 s. The Casimir-like effect decreases for larger D_i like the real Casimir effect. The solid line is drawn through the averages of the data points.

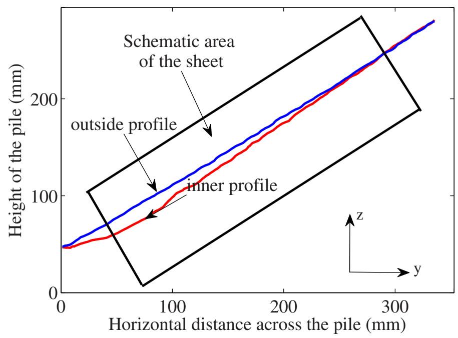

FIG. 6. (Color online) Slope profile of the rice pile averaged in x direction over 25 mm between the sheets (red lower curve) and directly outside the sheets (blue upper curve). Data shown are for the experiment with D_i=25 mm, for 8.64×10^4 s after start of the experiment.

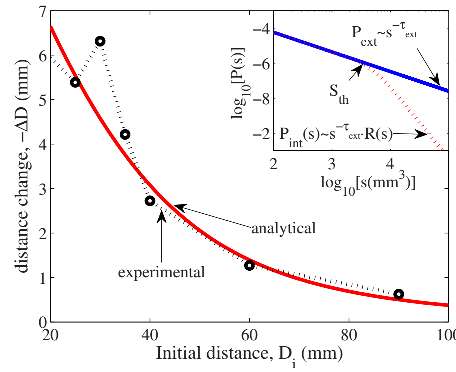

FIG. 7. (Color online) Analytical model (red solid line) and experimental results (black line with circles) of the Casimir-like effect for dependence of the distance change $-\Delta D$ on the initial distance D_i between the sheets. The experimental points correspond to the average values of Fig. 5. In the inset, the avalanche size probability densities $P(s)$ are shown: P_{int} for the region between the sheets for D_i=30 mm and P_{ext} for the outside region.

grains (≈7.0 mm). Even at D_i=20.0 mm there seems to be a small decrease in effect, which we attribute to "clogging" of the inner region: the motion of grains between the sheets is frustrated, making a decrease in D very hard.

C. Different profiles of the rice pile

From reconstruction of the three-dimensional shape of the rice pile, a comparison of the slope of the pile between and next to the sheets can be made. We find that always (independent of D_i and D), the slope between the sheets is steeper inside than outside the sheets. This effect is most prominent for small separation distances (for example, 20 mm). As an illustration, Fig. 6 shows the reconstructed profiles of the pile slope (averaged over D in the x direction) in the inner and outer regions after 24 h of the experiment for D_i=25 mm. The steeper slope between the sheets makes the pile in the inner area more unstable, which partly compensates for the stabilization due to the suppression of some of the avalanches. Due to this compensation, the system maintains its critical state.

IV. SIMPLE MODEL FOR THE DISTANCE DEPENDENCE OF THE CASIMIR-LIKE FORCE

We now formulate a simple model for the initial separation distance dependence of the force driving the sheets together. The forces acting on both sides of the sheets which keep them stable are neglected and are assumed to be of equal magnitude. Only forces due to the fluctuations, which are the avalanches on the pile of rice, will be considered. It was demonstrated before [16] that these fluctuations obey a power-law distribution of event sizes over more than three orders of magnitude:

$$P_{ext}(s) \sim s^{-\tau_{ext}}, \tag{1}$$

where s is the volume of an avalanche and $P_{ext}(s)$ is the probability density for observing an event of size s. It was found in Ref. [16] that τ_{ext}=1.12.

In our model, we assume that the distribution of event sizes is the same everywhere with the same exponent τ_{ext}. However, in the area between the sheets the avalanches with a size larger than some threshold s_{th} are suppressed. This assumption is based on analogy with the real Casimir effect, where fluctuations exceeding a certain wavelength are suppressed between the plates. Specifically, the avalanche size distribution in the inner region for this scenario can be represented by

$$P_{int}(s) \sim s^{-\tau_{ext}} R(s,s_{th}), \tag{2}$$

where $R(s,s_{th})$ is a cutoff function and s_{th} corresponds to the maximum avalanche size that fits between the sheets. Because the avalanches are rather compact fractal objects, we take for simplicity $s_{th}=\alpha LD$ as the maximum avalanche between the sheets. Here, α represents the average thickness of the fractal avalanche and L is the length of the sheet. The exact form of the cutoff function $R(s,s_{th})$ is not very important; here we take

$$R(s,D) \sim \frac{1}{1+\left(\dfrac{s}{\alpha LD}\right)^3}, \tag{3}$$

where α=0.65 mm and L=294 mm. Both $P_{ext}(s)$ and $P_{int}(s)$ are shown in the inset of Fig. 7.

In overdamped systems, like granular piles, it can be assumed that the velocity of the sheets is proportional to the force F acting on them [20]. By taking the integral over the time of the experiment we get that the total displacement ΔD is proportional to $\int F dt$. To proceed, it is assumed that the pile width is much greater than D such that the chances of an individual avalanche to reach the top of the inner and outer areas adjacent to the sheets are equal. Only avalanches traversing in the close vicinity of either side of the sheets can contribute to a force moving the sheets. The forces F_{int} and F_{ext} acting on the inner and outer sides of one sheet are taken proportional to the number of particles close to the sheets in such avalanches [occurring with the probabilities $P_{int}(s)$ and $P_{ext}(s)$, respectively]. Now we can express the integral of the resulting force over time as $\int F dt = \int (F_{int}-F_{ext}) dt$. This integral is over all possible avalanches that occur in the close vicinity of the sheets for the total duration of the experiment t_{exp}, which is 25 h for each experimental run. Hence,

$$\Delta D \sim \int_0^{t_{\mathrm{exp}}} F dt \sim \int_0^{s_{\mathrm{max}}} [sP_{int}(s) - sP_{ext}(s)]ds. \tag{4}$$

Here, s_{max} is the maximum size of an avalanche occurring in the close vicinity of a sheet (we take $s_{\mathrm{max}}=10^4$ mm^3). Since $\Delta D \ll D$ for $D_i \gtrsim 35$ mm we can substitute D with D_i, and finally we have

$$\Delta D = \beta \int_0^{s_{\mathrm{max}}} s^{1-\tau_{ext}}[R(s,D_i) - 1]ds. \tag{5}$$

In Fig. 7 the experimental distance dependence of ΔD on D_i is compared with Eq. (5). Since the proportionality constant between F and ΔD is unknown, we have fitted the constant β to the data, thus scaling the analytical curve to the experimental data. Clearly the analytical dependence is in agreement with the experimental data: the distance change ΔD decays very fast for increasing distance, in agreement with experiment. Similarly to the real Casimir effect, the decay of the experimental ΔD dependence can be fitted to a power law D_i^{γ} for $D_i \gtrsim 30$ mm; however, for this granular experiment $\gamma=-2.1$.

In conclusion, we constructed a simple analytical model by combining the known probability density function for avalanches with a cutoff function that suppresses avalanches that do not fit between the sheets. This simple model captures the essential experimental behavior.

V. DISCUSSION AND CONCLUSION

We have demonstrated above that a Casimir-like behavior in granular material is experimentally observed using two Plexiglas sheets positioned parallel to each other in the presence of fluctuations originating from rice avalanches. We now discuss correspondences and discrepancies with other Casimir-like effects.

Contrary to most other systems, ours displays a *spatial anisotropy*. Avalanches move primarily down the slope, although they spread across the entire slope. If we would have rotated the sheets by 90°, then the sheets would have been driven together trivially and not due to a Casimir-like effect. In the experimental geometry used here, no such bias or trivial effect and all signs of Casimir-like behavior are present, including the suppression of avalanches between the sheets (see Fig. 6). Moreover, we conducted an independent experiment showing that a continuous flow of rice grains down the slope does not result into a decrease in the separation distance between the sheets, which confirms that our observed forces are due to fluctuations in the rice pile.

Our system is *an out-of-equilibrium* system that tunes itself to stay in a critically fluctuating state (self-organized criticality). This means that in our system, like for the genuine Casimir effect, no tuning of external parameters is needed, contrary to effects based on fluctuations close to a second-order phase transition. The suppression of avalanches between the sheets even leads to a steeper slope, thus restoring the fluctuating state of the pile between the sheets.

Our system is *quasi-two-dimensional* since the avalanches affect predominantly the upper layers of rice in the pile. The Casimir-like effect we have observed is thus due to mainly two-dimensional fluctuations.

Due to the nature of a granular pile, our system is *strongly overdamped*, while the sheets are at least buried in the rice for 5 cm. This is contrary to most Casimir(-like) systems, where the sheets are free to move and the Casimir(-like) force can be easily measured by balancing with a known counterforce. The fact that we find that the two sheets tend to move together proves that on the average the forces on the sheets drive them together, even if the force itself is very difficult to determine. Since the sheets are partially buried in the rice, the lower part of the sheets is in principle less mobile than that close to or above the surface of the rice. In addition, the flow of rice during avalanches is mainly in the upper few centimeters of the pile. Hence, one would expect that the sheets would experience significant tilting during our experiments. Indeed we do find tilting, but it is rather limited (to about 1°).

In conclusion, we have experimentally demonstrated a Casimir-like force in a nonequilibrium system, i.e., a granular pile. The separation distance between two sheets was found to decrease over time, consistent with the effect of a net attractive force. For initial separation distances of 20.0 and 90.0 mm, the decreases after 10^5 s are 6.0 and 1.0 mm, respectively. A simple analytical model describes the observed behavior qualitatively rather well. Our system is more macroscopic than most Casimir-like systems studied so far, and we envision that similar observations may be made in other self-organized critical systems.

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